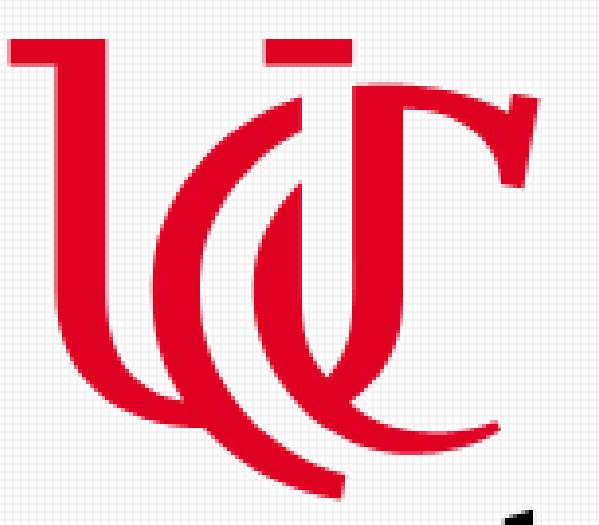


# Aqueous pigment dispersions: The thermodynamics of hierarchical aggregation

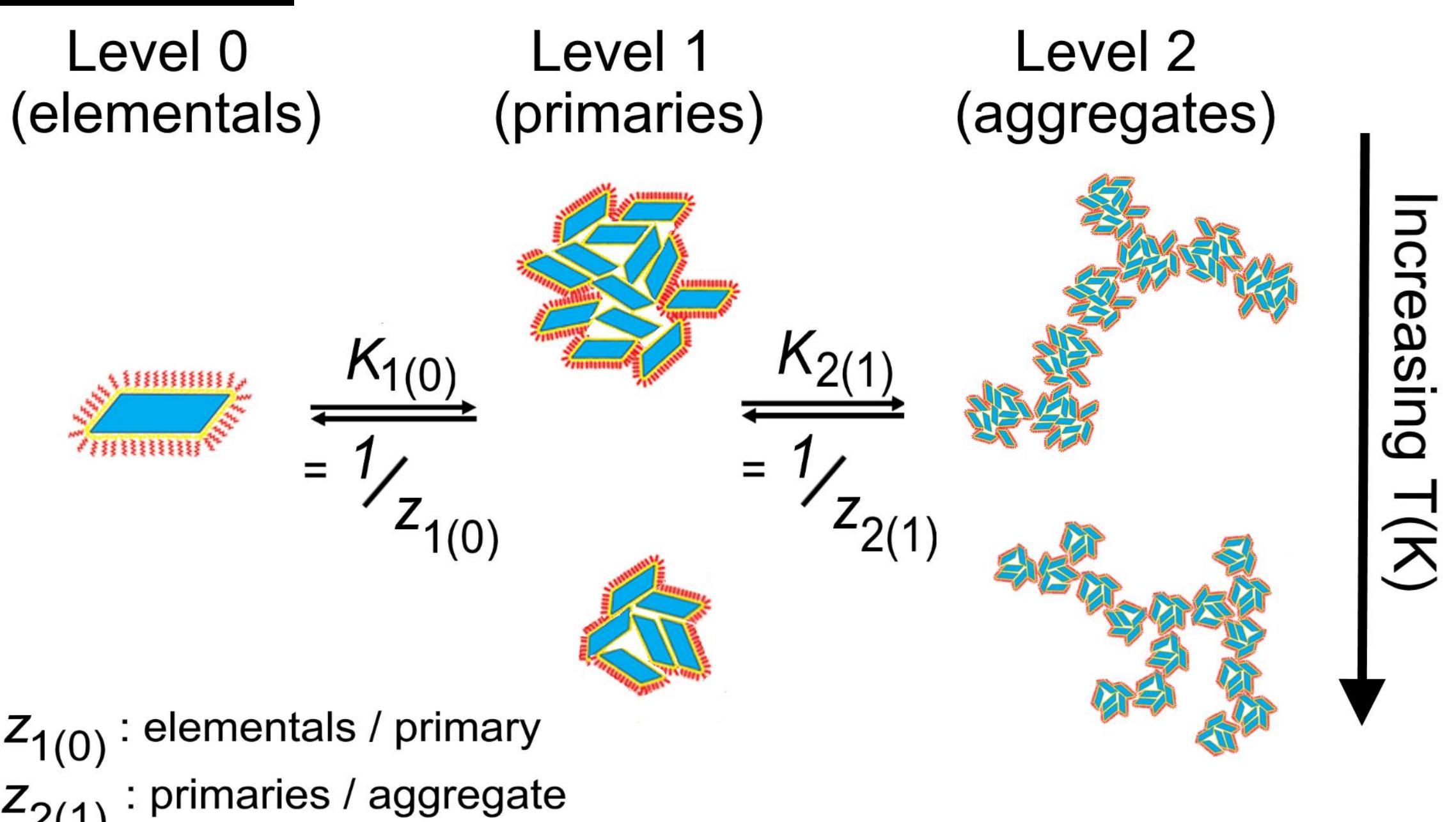


UNIVERSITY OF  
Cincinnati

Jianqi Wang<sup>a</sup>, Kabir Rishi<sup>a</sup>, Gregory Beaucage<sup>a</sup>

<sup>a</sup> Department of Materials Science & Engineering, University of Cincinnati, Cincinnati, OH 45221, USA

## Overview<sup>1</sup>



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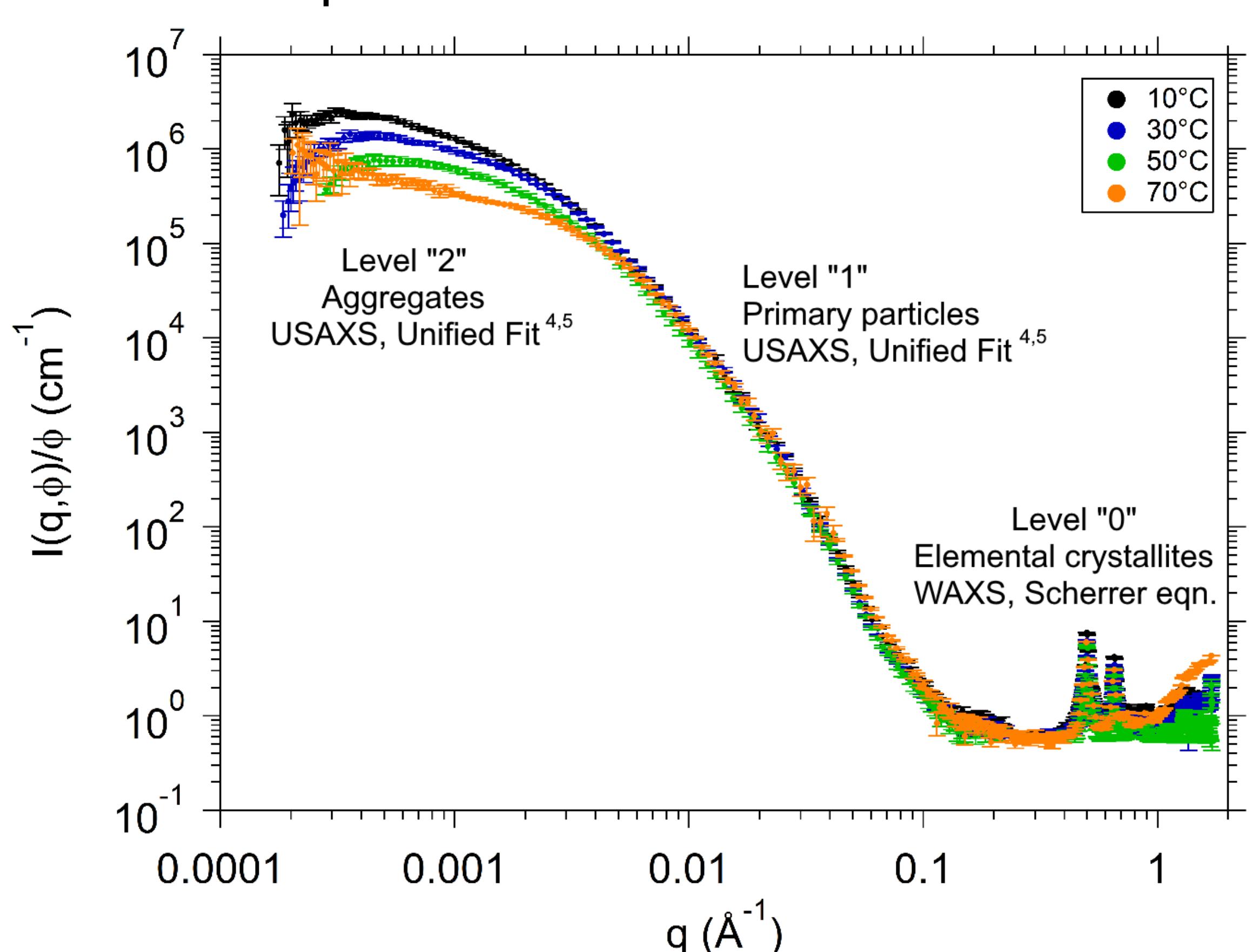
## Predictions of the Vogtt Model<sup>3</sup>

	$\Delta S_{1(0)}$	$\Delta S_{2(1)}$	$z_{1(0)}$	$z_{2(1)}$	prediction at elevated temperature
I	+	+	↓	↓	elemental particles increase at the expense of aggregates and primary particles due to a decreased entropic penalty to remove a subunit
II	-	-	↑	↑	aggregates and primary particles grow at the expense of elementals due to an increased entropic penalty for removal of a subunit
III	+	-	↓	↑	aggregates grow at the expense of the primary particle size
IV	-	+	↑	↓	primary particles grow at the cost of aggregate size

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## Methods

Surfactant (Triton X100) stabilized aqueous pigment dispersions (PY14 & PB15:3). USAXS / SAXS / WAXS at temperatures from 10°C to 80°C.



## Smoluchowski aggregation<sup>2,6</sup>

$$\frac{dn_k}{dt} = \frac{1}{2} \sum_{i+j \rightarrow k} C_{ij} n_i n_j - \sum_{i=1}^{\infty} C_{ik} n_i n_k$$

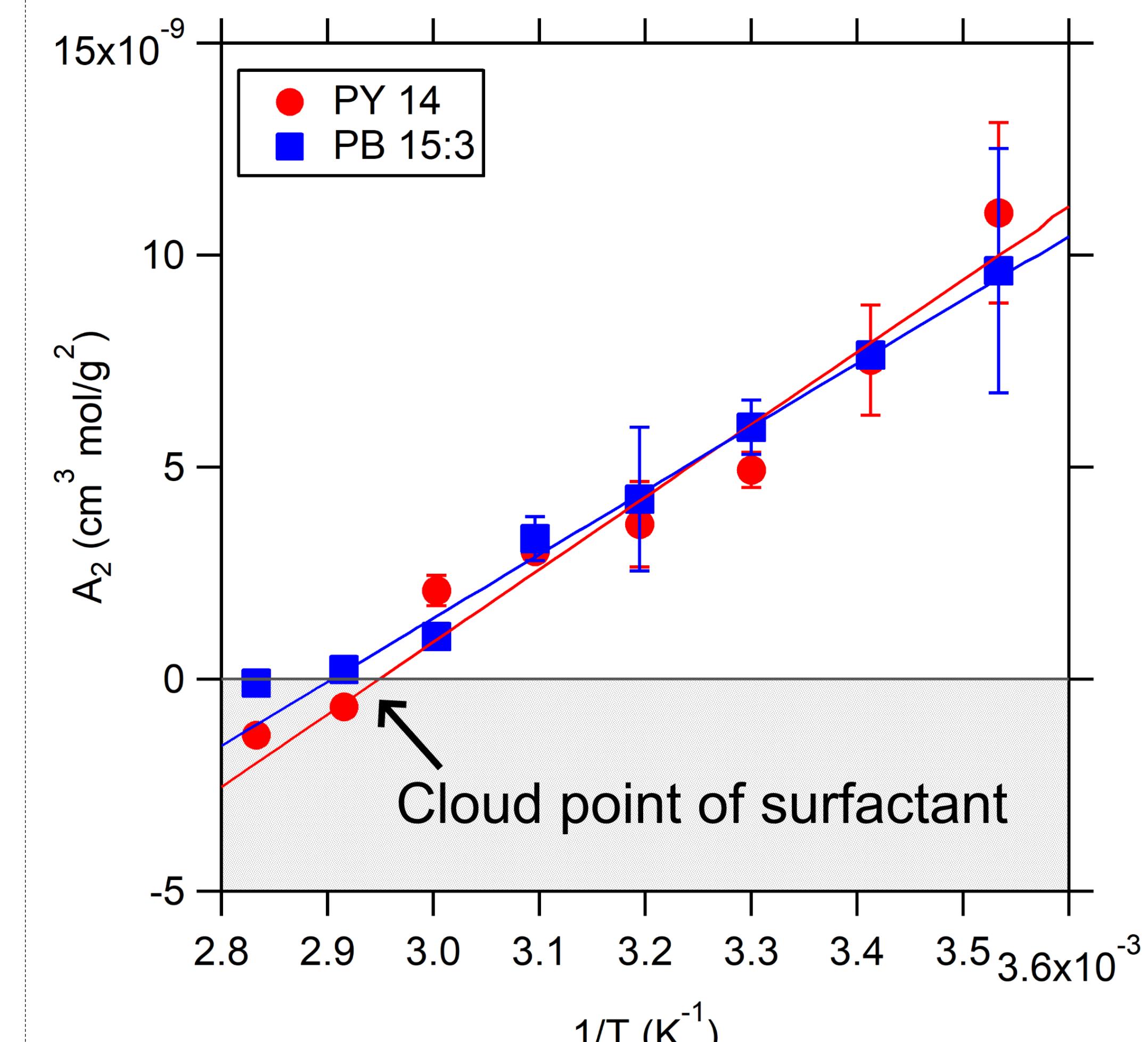
Formation of aggregates by any pair i and j  
Loss of aggregates due to aggregation

## New Vogtt Model<sup>3</sup>

$$\langle N_i(n_{i-1}) dn_{i-1} + z_{i(i-1)} dN_i(n_{i-1}) \rangle_t = 0$$

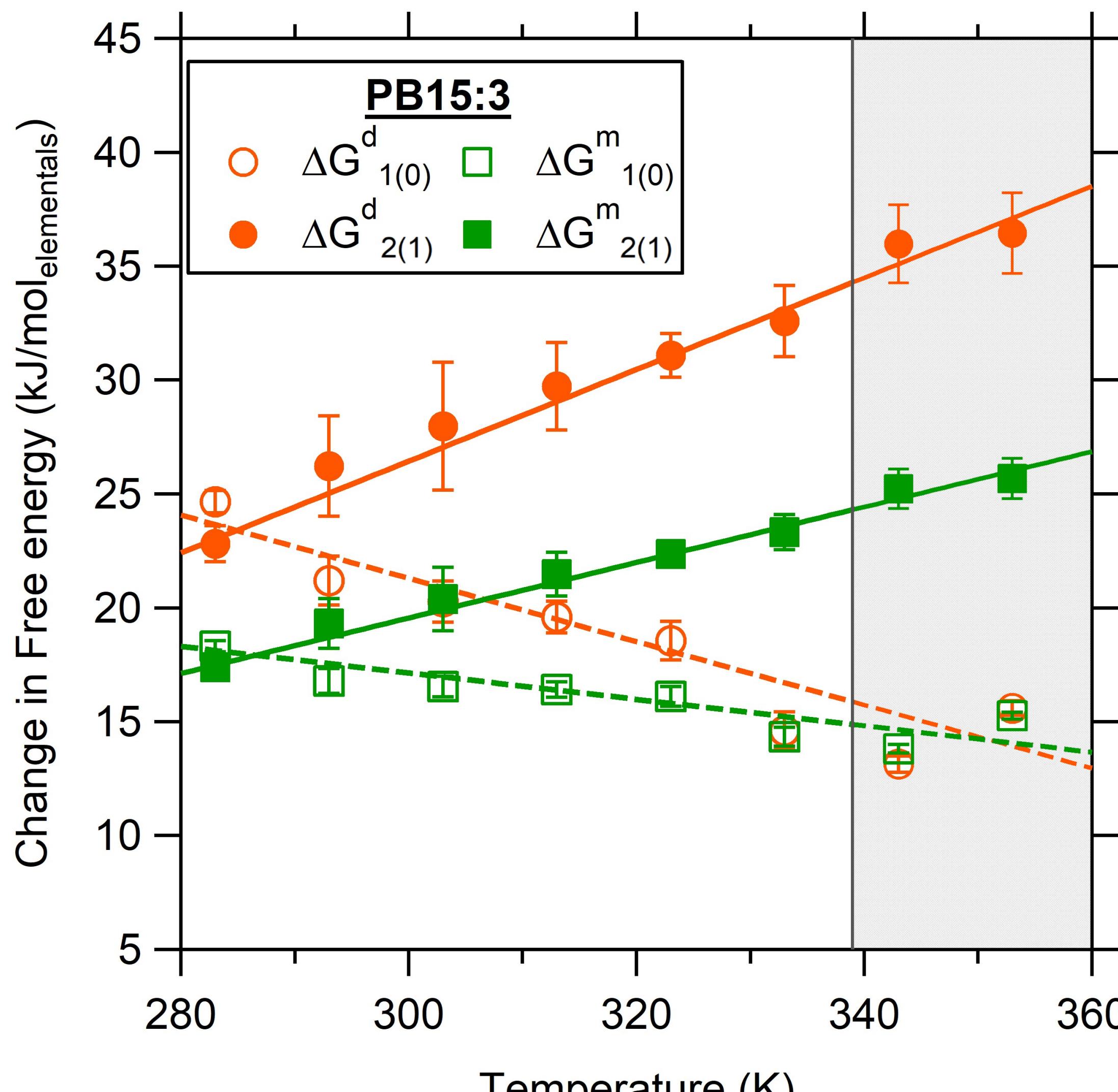
Change in the # of subunits  
Degree of aggregation

$$G_{i(i-1)} = RT \ln(z_{i(i-1)})$$

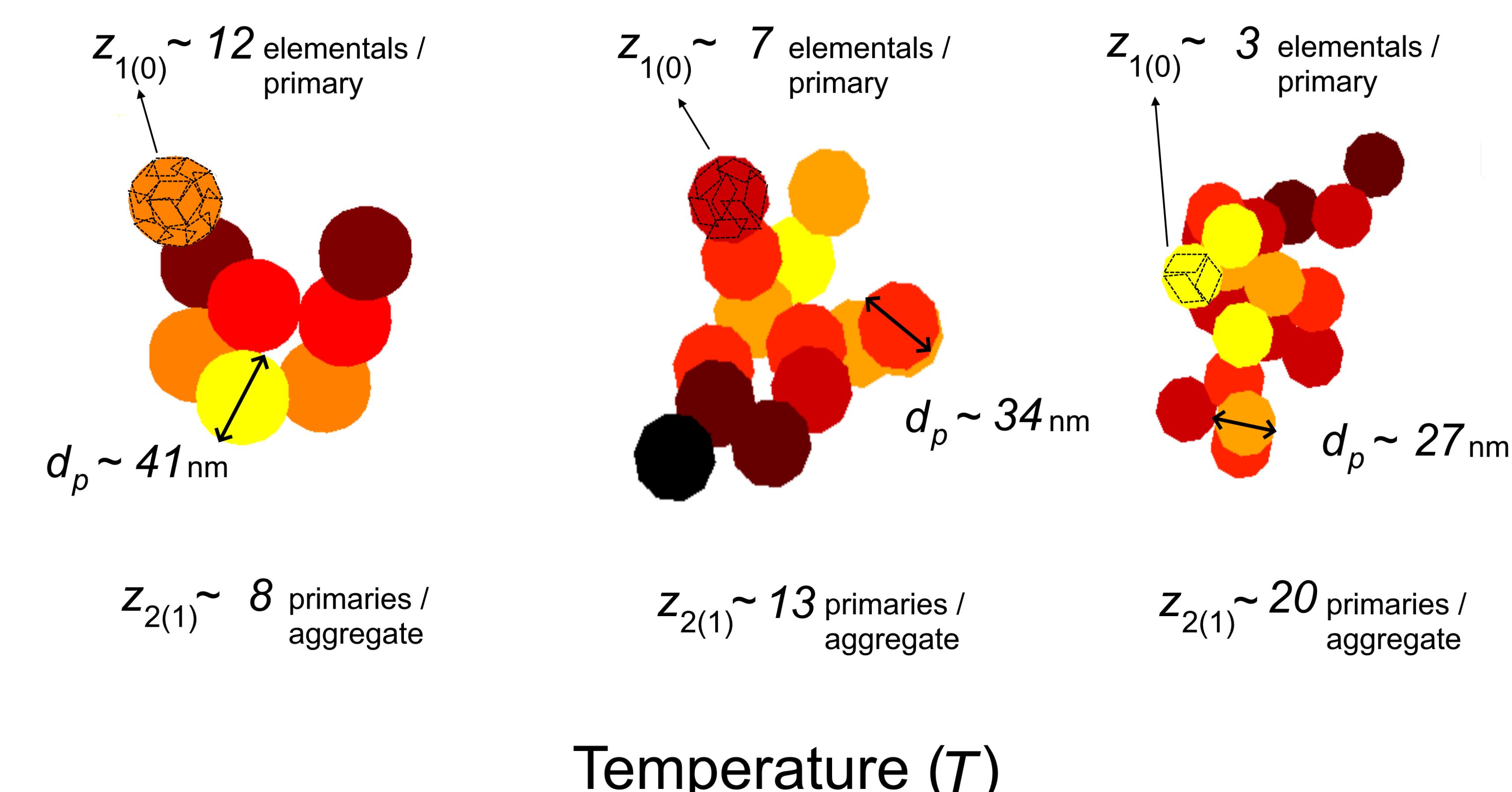


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## Thermodynamics of aggregation



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For further information, please contact:  
Gregory Beaucage beaucag@ucmail.uc.edu